Ordinary Differential Equation (ODE) - sample questions

Please pick only **one answer for each question**. Mark the answer you think is right. You do not need to write anything else on the sheet.

1. For a function $y(x)$, we denote $y' = \frac{dy}{dx}$ the derivative with respect to x. Given the ordinary differential equation

$$y' + y = e^{-x}$$

with initial condition $y(0) = 5$. Which of the following is the correct solution to this problem:

(a) $y(x) = (x - 5)e^{2x}$

(b) $y(x) = (x + 5)e^{-x}$

(c) $y(x) = e^{x} - e^{-x}$

Correct solution is (b).

2. Which of the following equations is a second-order, linear ODE:

(a) $\frac{dy}{dt} = y + 1$

(b) $\frac{d^2y}{dt^2} + y\frac{dy}{dt} + y = 1$

(c) $\frac{d^2y}{dt^2} + t^3 y = 0$

(d) $\frac{d^3y}{dt^3} + y\frac{dy}{dt} = 1$

Correct solution is (c).

3. Consider the first-order, autonomous equation

$$\frac{dy}{dt} = 1 - y$$

Without solving it (e.g. you could sketch the graph of $f(y) = 1 - y$), what are the equilibrium points of the system and their classification?

(a) $y = -1$ as stable point.

(b) $y = 1$ as stable point.

(c) $y = 1$ as saddle point.

(d) $y = 1$ as unstable point.

Correct solution is (b).

4. Given the differential equation

$$y'(t) = 2t - 5$$

with initial condition $y(1) = 4$. We want to solve it numerically with Euler's method. What is the formula applied to this problem?

(a) With h being a time step, $t_n = t_{n-1} + h$ and $y_{n+1} = y_n + h(2t_n + 5)$ for $n = 1, 2, ..$; starting with $t_0 = 1$ and $y_0 = 4$.

(b) With h being a time step, $t_n = t_{n-1} + h$ and $y_{n+1} = y_n + ht_n$ for $n = 1, 2, ..$; starting with $t_0 = 0$ and $y_0 = 4$.

(c) With h being a time step, $t_n = t_{n-1} + h$ and $y_{n+1} = y_n + h(2t_n - 5)$ for $n = 1, 2, ..$; starting with $t_0 = 0$ and $y_0 = 4$.

(d) With h being a time step, $t_n = t_{n-1} + h$ and $y_{n+1} = y_n + h(2t_n - 5)$ for $n = 1, 2, ..$; starting with $t_0 = 1$ and $y_0 = 4$.

Correct solution is (d).