

List of Topics for Ph.D. Qualifying Examination

(Updated: June 2021)

MATHEMATICS

There are five areas within mathematics, as defined below. The student should be prepared to answer questions from *three* areas of their choice. Each area contains a list of topics and a list of courses to indicate the depth of these topics. However, the exam is not intended to be a final exam in any of these courses; instead, the exam evaluates the general mathematical understanding of these areas.

1. **Linear algebra:** finite dimensional vector spaces, linear transformations and matrices, eigenvectors and eigenvalues, positive definiteness, diagonalization, singular value decomposition, operators on infinite dimensional spaces, principal component analysis, applications to differential and integral equations.
2. **Complex variables:** analyticity, integration, series, singular points, residues and applications, conformal mapping, analytic continuation, Cauchy residual theorem, Taylor and Laurent series.
3. **Ordinary differential equations:** linear systems, initial- and boundary-value problems, singular points, Fourier and Laplace transforms, phase plane methods, linearized stability Sturm-Liouville theory.
4. **Partial differential equations:** type and normal forms for second order equations, qualitative properties for elliptic, hyperbolic and parabolic equations, well-posed problems, separation of variables, transform methods, method of characteristics, fundamental solutions, Green's function, shock waves.
5. **Numerical Analysis:** Interpolation, numerical integration and differentiation, modified equation analysis, dispersion vs. dissipation error, ODE integration schemes, familiarity with linear solvers.

FLUID MECHANICS

The student should be prepared to answer questions from any of the following topics.

1. **Kinematics:** Eulerian and Lagrangian description, fluid deformation, rate of strain, shear, dilatation, vorticity, circulation, material path lines, streaklines, streamlines.
2. **Conservation laws:** for mass, momentum, energy (control volume, differential form).
3. **Euler equations of motion**
4. **Constitutive relations:** Newtonian fluids, Navier-Stokes equations.
5. **Potential flow:** velocity potential, Kelvin's theorem, d'Alembert's paradox, Bernoulli's equation, complex potential, Blasius' theorems.
6. **Boundary layers:** scaling, laminar boundary layers on flat plates and free shear layers (mixing layers, wakes, and jets).
7. **Stability:** general concepts, instabilities of parallel shear flows (viscous and inviscid).
8. **Turbulent flow:** general characteristics, Reynold's averaging, scaling of turbulent boundary layers (e.g., law of the wall)
9. **Flow over bluff bodies:** drag, separation, wakes.
10. **Gasdynamics:** stagnation conditions/properties, normal/oblique shocks, Prandtl-Meyer expansions, simple waves, quasi-one-dimensional flow.
11. **Free surfaces:** surface tension, small amplitude surface gravity waves, dispersion.

SOLID MECHANICS

The student should be prepared to answer questions from any of the following topics.

1. **Fundamentals:** Finite deformation, deformation gradient, 3-D displacement, small strain theory, compatibility, equilibrium equation, traction, stress, boundary conditions, constitutive equations.
2. **Linear elasticity:** Basic equations, generalized Hooke's law, plane strain and plane stress, axisymmetric problems, 3-dimensional problems, reciprocal theorem, transformation of stress and strain, St. Venant's principle, thermal effects, thick tubes, Kelvin's point/line load problem, stress concentration, torsion of non-circular and thin-walled cross sections, theory of beams, rods, cables.
3. **Energy methods:** Principle of virtual work, theorem of potential energy, basics of the finite element method, discretization, concepts of nodal degrees of freedom and shape function, stiffness matrix and boundary conditions.
4. **Theory of plates and shells:** Kirchhoff theory of plates, membrane theory of shells.
5. **Elastic buckling of columns:** Column buckling, beam-column theory, Euler loads.
6. **Composites:** Theory of fiber reinforced composite, micromechanical models, effective moduli, ply and laminate mechanics.
7. **Plasticity and viscoelasticity:** Rigid-plastic behavior, ideal plasticity and strain hardening, yield functions and flow rules, plastic bending, Maxwell and Voigt solids, creep and relaxation functions.
8. **Fracture mechanics:** Crack tip fields, stress intensity factor, energy release rate, J-integral, fracture toughness.

THERMAL SCIENCES

The student should be prepared to answer questions from any of the topics listed under “heat transfer” and “thermodynamics”.

Heat Transfer

1. **Conservation laws** and heat diffusion equation
2. **1-D Steady-State Conduction** (Steady-state 1-D conduction with and without generation, Numerical solutions to steady-state 1-D conduction problems, Analytical solutions for constant cross-section extended surfaces).
3. **2-D Steady-State Conduction** (Shape factors, Separation of variable solutions, Superposition, Numerical solutions to steady-state 2-D conduction problems).
4. **Transient Conduction** (Analytical and numerical solutions to 0-D transient problems, Semi-infinite 1-D transient problems and self-similar solution, Separation of variables for transient problems)
5. **External Forced Convection** (Laminar boundary layer concepts, Boundary layer equations, Dimensional analysis in convection, Self-similar solution for Laminar flow over a flat plate, Turbulent boundary layer concepts, Reynolds averaged equations, Laws of the wall, Integral solutions, External flow correlations)
6. **Internal Forced Convection** (Internal flow concepts, Internal flow correlations, Energy balance for internal flows, Analytical and numerical solutions for internal flows)
7. **Natural Convection** (Natural convection concepts, Natural convection correlations)
8. **Thermal Radiation** (Radiation concepts, Emission of radiation by a blackbody, Radiation exchange between black surfaces, Radiation characteristics of real surfaces, Diffuse gray surface radiation exchange, Radiation with other heat transfer mechanisms, The Monte Carlo method).
9. **Heat Exchangers** (Concepts and types of heat exchangers, The LMTD method, The effectiveness-NTU method, Heat exchangers with phase change, Design and performance calculations, Regenerators)
10. **Mass Transfer** (Mass diffusion and Fick's law, Transient diffusion through a stationary medium, Mass convection, Simultaneous heat and mass transfer, Mass diffusion with chemical reactions).

Thermodynamics

1. **Intensive and extensive variables**; definitions of temperature, pressure and chemical potential.
2. **Equilibrium** (thermal, mechanical, and chemical);
3. **Thermodynamic potentials**, canonical variables, min/max principles.
4. **First and second law of thermodynamics**
5. **Entropy generation**, exergy destruction, exergy analysis
4. **Work and heat**: quasi-static and reversible processes, irreversibility, heat flow, maximum work theorem. Carnot cycle, heat pumps.
5. **Formal relations**: Euler equations, Gibbs-Duhem equation, Maxwell relations.
6. **Multiphase systems**: latent heat, Clausius-Clapeyron equation, van der Waals phase transition, continuity of the vapor and liquid states, Andrews diagram, Gibbs phase rule, corresponding states, critical point phenomena.
7. **Chemically reactive systems**: equilibrium, chemical potential, reversible / irreversible reactions, steady-flow combustion, constant-volume combustion.
8. **Power generation**: maximum power from hot stream, external irreversibilities, internal irreversibilities, advanced steam-turbine power plants, advanced gas-turbine power plants, combined steam/gas plants.
9. **Statistical mechanics**: fundamental postulates, statistical definition of entropy, microstates and microcanonical and canonical ensembles, partition functions and their use to calculate thermodynamics properties.

DYNAMICS and CONTROL

The student should be prepared to answer questions from any of the following topics.

Dynamics

1. **Phase Space:** Phase space, phase portraits, linear oscillators.
2. **Equilibrium solutions (continuous systems):** Fixed points, stability concepts, linearization and stability analysis, bifurcation types.
3. **Equilibrium solutions (maps):** Fixed points, stability concepts, linearization and stability analysis, bifurcation types.
4. **Periodic solutions:** Bifurcations, shooting technique, Poincare section, quasi-periodic solutions.
5. **Topics in nonlinear oscillations:** Scaling and normalization, the method of multiple scales, the Duffing equation, primary and secondary resonances, self-excitation, parametric excitation.
6. **Basic Principles of Linear Dynamics:** Newtonian dynamics, and energy approaches.
7. **Basic Principles of Linear Vibrations:** Free and forced vibrations of discrete systems, eigenvalue problems

Control

1. **Modeling and analysis of feedback systems:** use of ordinary differential equations for modeling mechanical systems. Stability of equilibrium points and linearization. Input/output modeling and performance specifications (frequency and step response).
2. **Classical control theory:** control of single-input/single-output systems. State space control and state feedback, including controllability and observability. Loop analysis of feedback systems using Nyquist and Bode plots. Design of feedback compensators using loop shaping techniques, including PID control.
3. **Modern feedback control design:** stability and performance analysis in frequency and time domain. Optimal control and linear quadratic regulators.