Basic Calculus - sample

1. Calculate explicitly the function $f(x)=\sum_{n=0}^{N} e^{-xn}$

(a) $f(x)=\frac{1-e^{-xN}}{1+e^{-x}}$

(b) $f(x)=\frac{1+e^{-x(N+1)}}{1-e^{-x}}$

(c) $f(x)=\frac{1-e^{-x(N+1)}}{1-e^{-x}}$

(d) $f(x)=\frac{1-e^{-xN}}{1-e^{-x}}$

The correct answer is (b)

2. Choose the correct derivative of $f(x)=\frac{1}{2}\ln(\tan x)$

(a) $f'(x)=\frac{1}{\sin 2x}$

(b) $f'(x)=\frac{1}{\tan 2x}$

(c) $f'(x)=\frac{1}{\cos 2x}$

(d) $f'(x)=\tan 2x$

The correct answer is (a)

3. What is the correct limit of $\lim_{x\to 0}\frac{\ln(x^2+x+1)}{x+2x^2}$

(a) 0

(b) $+\infty$

(c) 1

(d) $\frac{1}{2}$

The correct answer is (c)

4. Using Taylor's expansion, expand the function $f(x)=e^{(1-x)^2}$ at $x=1$ to the second order in *x*.

(a) $1+x+\frac{1}{2}x^2$

(b) $1+(1-x)+\frac{1}{2}(1-x)^2$

(c) $2-2x+x^2$

(d) $-2(1-x)e^{(1-x)^2}$

The correct answer is (b)

5. Find the exponential form of

$$\left(\frac{-4}{\sqrt{3}+i}\right)^{19}$$

(a) $-e^{19i\pi/6}$

(b) $-4^{19}\left(e^{5i\pi/6}\right)^{19}$

(c) $2^{19}e^{95i\pi/6}$

(d) $-4^{19}e^{95i\pi/6}$

The correct answer is (c)

6. What can you say about the function $f(x)=|x|$

(a) $f(x)$ and $f'(x)$ are both discontinuous

(b) $f(x)$ is discontinuous and $f'(x)$ is continuous

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(d) $f(x)$ is continuous and $f'(x)$ is discontinuous

The correct answer is (d)