Linear Algebra - sample questions

Please pick only **one answer for each question**. Mark the answer you think is right. You do not need to write anything else on the sheet.

1. What is the **rank** of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) Matrix $\mathbf{A}$ has $\text{rank}(\mathbf{A}) = 1$.

(b) Matrix $\mathbf{A}$ has $\text{rank}(\mathbf{A}) = 2$.

(c) Matrix $\mathbf{A}$ has $\text{rank}(\mathbf{A}) = 3$.

Correct solution is (b)

2. The inverse of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$$

is denoted by $\mathbf{A}^{-1}$. Using the inverse formula, $\mathbf{A}^{-1}$ is

(a)
$$\begin{bmatrix} 1/5 & -2/5 & 1/5 \\ 7/15 & 1/15 & -1/5 \\ -2/15 & 4/15 & 1/5 \end{bmatrix}$$

(b)
$$\begin{bmatrix} -1/5 & -2/5 & -1/5 \\ 7/5 & 1/5 & -1/5 \\ 2/5 & 4/5 & 1/5 \end{bmatrix}$$

(c)
$$\begin{bmatrix} -1 & -2 & -1 \\ 1 & -1 & -2 \\ -2 & 0 & -3 \end{bmatrix}$$

Correct solution is (a)

3. Find all solutions to the following system of equations using **Cramer's Rule**:

$$\begin{aligned} x_1 + x_2 + x_3 &= 3 \\ -x_2 + 2x_3 &= 1 \\ -2x_1 + x_3 &= -1 \end{aligned}$$

This system in matrix form $\mathbf{A}\,\mathbf{x} = \mathbf{b}$

(a) has a 3×3-matrix $\mathbf{A}$ with a zero determinant

(b) has a determinant $det(\mathbf{A}) = 2$

(c) has a replacement matrix

$$\mathbf{A}_i = \begin{bmatrix} 3 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \text{ for } i = 2$$

.

(d) has the solutions $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

Correct solution is (d)

4. Let T: $\mathcal{R}^3 \to \mathcal{R}^3$ be the linear transformation defined by

$$(x_1, x_2, x_3) \mapsto (2x_1 + x_2,\ x_2 - x_3,\ x_2 + 3x_3)$$

This can be written in matrix form $T(x_1, x_2, x_3) = \mathbf{A}\,\mathbf{x}$ where the components of matrix $\mathbf{A}$ define the linear transformation. The transformation T is ..

(a) associated with a matrix $\mathbf{A}$ which has a determinant $det(\mathbf{A}) = 8$.

(b) not bijective.

(c) not isomorphic.

Correct solution is (a)