Linear Algebra - sample questions

Please pick only **one answer for each question**. Mark the answer you think is right. You do not need to write anything else on the sheet.

**1.** What is the **rank** of the following matrix

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{bmatrix}
\]

(a) Matrix \(A\) has \(\text{rank}(A) = 1\).
(b) Matrix \(A\) has \(\text{rank}(A) = 2\).
(c) Matrix \(A\) has \(\text{rank}(A) = 3\).

**Correct solution is (b)**

**2.** The inverse of the following matrix

\[
A = \begin{bmatrix}
1 & 2 & 1 \\
-1 & 1 & 2 \\
2 & 0 & 3
\end{bmatrix}
\]

is denoted by \(A^{-1}\). Using the inverse formula, \(A^{-1}\) is

(a) \[
\begin{bmatrix}
1/5 & -2/5 & 1/5 \\
7/15 & 1/15 & -1/5 \\
-2/15 & 4/15 & 1/5
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
-1/5 & -2/5 & -1/5 \\
7/5 & 1/5 & -1/5 \\
2/5 & 4/5 & 1/5
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
-1 & -2 & -1 \\
1 & -1 & -2 \\
-2 & 0 & -3
\end{bmatrix}
\]

**Correct solution is (a)**
3. Find all solutions to the following system of equations using Cramer's Rule:

\[ \begin{align*}
  x_1 + x_2 + x_3 &= 3 \\
  -x_2 + 2x_3 &= 1 \\
  -2x_1 + x_3 &= -1
\end{align*} \]

This system in matrix form \( A \mathbf{x} = \mathbf{b} \)

(a) has a 3 \( \times \) 3-matrix \( A \) with a zero determinant

(b) has a determinant \( \det(A) = 2 \)

(c) has a replacement matrix

\[
A_i = \begin{bmatrix}
  3 & 1 & 1 \\
  1 & -1 & 2 \\
  -1 & 0 & 1
\end{bmatrix}
\text{ for } i = 2
\]

(d) has the solutions \( x_1 = 1, x_2 = 1, x_3 = 1 \)

**Correct solution is (d)**

4. Let \( T: \mathbb{R}^3 \to \mathbb{R}^3 \) be the linear transformation defined by

\[ (x_1, x_2, x_3) \mapsto (2x_1 + x_2, x_2 - x_3, x_2 + 3x_3) \]

This can be written in matrix form \( T(x_1, x_2, x_3) = A \mathbf{x} \) where the components of matrix \( A \) define the linear transformation. The transformation \( T \) is ..

(a) associated with a matrix \( A \) which has a determinant \( \det(A) = 8 \).

(b) not bijective.

(c) not isomorphic.

**Correct solution is (a)**